Human Capital Formation, Inequality, and Competition for Jobs*

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Abstract

This paper develops a model where heterogeneous agents compete for the best available jobs. Firms, operating with different technologies, rank job candidates in the human capital dimension and hire the best available candidate due to complementarities between the worker's human capital and technologies used in the production process. As a result, individuals care about their relative ranking in the distribution of human capital because this determines the firm they will be matched with and therefore the wage they will receive in equilibrium. The paper rationalizes a different channel through which peer effects and human capital externalities might work: competition between individuals for the best available jobs (or prizes associated with the relative position of individuals). We show that more inequality in the distribution of endowments negatively affects aggregate efficiency in human capital formation as it weakens competition for jobs between individuals. However, we find that the opposite is true for wage inequality, namely, more wage inequality encourages competition and, as a result, agents exert more effort and accumulate more human capital in equilibrium.

Keywords: Human Capital, Inequality, Competition, Relative Ranking.

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1. Introduction

This paper develops a model of human capital accumulation and competition for jobs where there are strategic interactions between heterogeneous agents that compete for job positions with different wages. We argue that higher inequality in the distribution of the endowments necessary to accumulate human capital negatively affects aggregate efficiency in human capital formation. This effect is beyond the standard Jensen's inequality channel because inequality also affects individuals' incentives to accumulate human capital when they confront competition from their close peers for the best available jobs. Intuitively, as the mass of "close" competitors for job positions with different compensations increases, the incentives to differentiate from each other, by exerting higher effort and accumulating more human capital, also increases. However, more wage inequality (i.e. more inequality in the returns to human capital accumulation) has the opposite effect, fostering competition for job positions between individuals, and, by doing so, increasing aggregate efficiency in human capital formation. In equilibrium, individuals' optimal choices depend on both, the distribution of endowments that are complementary to time and effort invested in the accumulation of human capital (the distribution of opportunities), and on the distribution of wages (the distribution of returns to human capital accumulation). As we will show, changes in the degree of inequality in each of these distributions have opposite effects on individuals' choices and on aggregate efficiency in human capital formation.

On the demand side of the labor market, we will assume that firms, operating with different technologies, rank individuals in the human capital dimension and hire the best available candidate due to complementarities between human capital and technologies in the production process. On the supply side, in choosing the optimal level of investment in the accumulation of human capital, individuals take two effects into account when evaluating the marginal benefit from exerting effort in the accumulation of human capital. The first effect is the usual direct marginal increase in income that results from a marginal increase in human capital (as in a standard model *á-la*-Becker (1964)). The second effect comes from the marginal change in the relative position of the individual that results when she invests one extra unit of time and effort in the accumulation of human capital, which, in turn, determines her relative position in the human capital distribution and the wage she will receive in equilibrium. As a result of this last effect, there is a so-called "rat-race" where individuals try to out-compete other individuals for the best available jobs. Although this rat-race might create excessive competition for jobs, the level of investment is socially optimal because individuals receive the full returns of their investment in human capital accumulation. That is, we will assume that there is a perfectly competitive labor market where individuals' human capital is remunerated according to its marginal product. Although more effort is exerted in equilibrium when individuals compete with each other for the best available job positions than in a standard model where there is no competition, this so-called 'excessive competition' is fully efficient because the pricing of human capital in the labor market fully compensates individuals for their extra investment. In fact, when making the optimal decision on the amount of investment in human capital, individuals trade-off the disutility from exerting extra effort in the competition for jobs for the greater utility they obtain from being able to match with firms that operate with better technologies and, hence, pay higher wages. If labor markets were not fully competitive and, for instance, wages were determined by Nash bargaining between firms and employees, then the excessive competition would not be fully efficient.[1] However, this result relies on strong assumptions about the pricing of human capital in the labor markets.

Our model assumes that individuals' concerns for relative ranking are instrumental. That is, individuals care about their relative position in the distribution of human capital not because they derive utility from relative ranking *per se*, but because their relative position determines the wage they will receive in equilibrium.

The literature on how inequality affects human capital formation has focused mostly on the role of credit market imperfections, wherein relatively poor individuals face financial constraints to pay for the costs associated with human capital accumulation, as they cannot use future earnings as collateral for the loans necessary to cover these costs. Furthermore, if there are decreasing returns to human capital accumulation, it is precisely these individuals (the poor) who have the largest returns to resource investments in education. As a result, a redistribution of resources from rich to poor individual would increase aggregate efficiency in the accumulation of human capital because of the reallocation of resources towards more profitable investments. This theoretical idea has been extensively developed in the literature since the work of Galor and Zeira (1993) and Banerjee and Newman (1993). Other developments have been proposed by De Gregorio (1996) and Bénabou (1996, 2000).[2] Empirical evidence has been found in favor of the hypothesis that inequality affects human capital accumulation in the presence of credit constraints (see Flug et al., 1998 and De Gregorio, 1996). In a recent paper, Mejía and St-Pierre (2007) show that inequality in the endowments that are complementary to effort in the schooling process (inequality of opportunities) affects aggregate efficiency in the accumulation of human capital without

[1] See Moen (1999).

[2] See Aghion et al. (1999) for a throrough review of the literature.

relying on credit market imperfections. The argument in that paper is that there are crucial complementary factors to the schooling process that are non-purchasable when the time for making investment decisions in education comes (i.e. parental schooling level, pre and post natal care, etc.). Because there are decreasing returns to time investment in human capital accumulation, and time investment in education is complementary to these factors, more inequality negatively affects aggregate human capital. Other papers in the literature have also explored political economy channels through which inequality affects human capital formation and economic growth. In particular, Glomm and Ravikumar (1992) and Ferreira (2001) emphasize the choice of public versus private schooling made through a political process as a key determinant of how inequality affects human capital formation (see Glomm and Ravikumar, 1992, and Ferreira, 2001).

The main contribution of this paper is to provide a rationale for a new, perhaps complementary, channel through which the inequalities of endowments and returns affect the incentives for human capital accumulation. An important difference with the existing literature is that the model we propose here includes strategic interactions between individuals. That is, an individual's return from the accumulation of human capital depends not only on his own choices and on the production technologies, but also on the entire distribution of endowments and returns. In other words, we argue that in deciding the optimal investment in human capital formation, there are strategic interactions between individuals. In this respect our model is also related to existing works on human capital externalities, and to the literature on peer effects in education. While most of the empirical literature on peer effects has focused on the effect of average education of peers on different measures of educational attainment of each student in a given class (that is, on linear-in-means peer effects), two recent papers find that, in fact, the structure of peer effects is highly non-linear. That is, students benefit differently from the inclusion of a new student in the class depending on their relative position in the class and the relative position of the entering student. In particular, students benefit significantly more from the inclusion in their class of new students that are similar to them (see Hoxby and Weingarth, 2007, and Ding and Lehrer, 2006), just as our model would predict. Human capital externalities have also been modeled in the literature as an average mean effect, that is, it is average human capital in the economy that affects each individual's marginal productivity in production (Lucas, 1988). In the existing literature on peer effects and human capital externalities individuals benefit from being close to more educated students or colleagues because of close collaboration and spillovers in the classroom or in the workplace. In this paper there are two important novel things relative to the literature just described. First, individuals are affected differently from an entering student in their cohort depending on their relative position and the relative position of the entering student. In particular, an individual is affected more by the choices made by those individuals close to her in the distribution than by the choices of individuals who are very different (as was shown empirically by Hoxby and Weingarth, 2007, and Ding and Lehrer, 2006). And, second, we argue that individuals are affected by other individuals not because of close collaboration and spillover effects in the classroom or the workplace but because they are competing with each other for the best available jobs. While our model does not rule out important effects due to collaboration and cooperation, we do propose another potentially important way through which peer effects or human capital externalities might work: competition for the best available jobs (or other relevant prizes associated with relative position). Thus, our paper has important implications (predictions) for the empirical literature on peer effects and human capital externalities. Namely, we argue that in measuring human capital externalities or peer effects one should not only account for the mean human capital in the population but, also, for higher moments of the distribution of education. In particular, human capital externalities (due to competition) should be larger in societies with less inequality of opportunity and, also, in those parts of the distribution of endowments with a greater mass of individuals. Also, the model predicts that peer effects and human capital externalities associated with competition should be larger in environments where the prizes associated with the relative position in the final dimension (grades, achievements, etc.) are more differentiated.

In addition to this introduction, the paper contains four sections. Section 2 discusses how concerns for relative position have been introduced in the economic literature and presents a short review of related contributions. In Section 3 we present the simple version of the model with two individuals and two firms. In section 4 we develop the general model. Section 5 concludes.

2. Concerns for relative ranking in the economics literature

Since the seminal work of Thorstein Veblen (1899), *A Theory of the Leisure Class*, several economists have argued that concerns for status (or the relative position in some relevant dimension(s)) have important economic consequences.[3] A central discussion in the literature that deals with concerns for relative ranking has to do with how we should understand such concerns, that is, whether they are direct or instrumental. While in the former case

[3] The reader is refered to Bastani (2007) for a thorough review of the literature on concerns for relative ranking.

people have concerns for status because they obtain utility from having high status in its own sake, in the latter people care about status because status directly affects the goods and services that individuals ultimately consume (Postlewaite, 1998). The strongest argument for incorporating direct concerns for relative position is an evolutionary one, the case for not incorporating direct concerns for status in the utility function is that economic models that incorporate them typically allow for very diverse behavior, there are almost no restrictions on equilibrium behavior and, as a result, the models lose predictive power.[4]

Most of the contributions that have emphasized the importance of concerns for relative ranking have focus on conspicuous consumption. The idea is the following: because wealth is unobservable, the consumption of conspicuous goods serves as a signal of non observable ability. Furthermore, if there are complementary interactions between individuals (for instance, at the household level between men and women, or at the workplace between employees and employers) conspicuous consumption might be welfare enhancing, even when the costs of conspicuous consumption[5] are taken into account, as they allow for a better (more efficient) matching (among others, see Cole et al., 1992 and 1995, Bagwell and Bernheim, 1996, and Rege, 2000). While concerns for status might generate excessive competition, this does not mean that excessive competition is inefficient (as has been argued by Frank, 1999 and others). In fact, when status can be purchased in a competitive market, the cost of acquiring status is simply a transfer payment that adds to the seller's wealth. For instance, Becker and Murphy (2000, ch. 4) show that competition for mates is fully efficient if the value that someone brings to the marriage is fully priced. In the same book, Becker, Murphy and Werning take Frank's (1999) example of wearing high heels and argue that "the demand for high heels is efficient, even when such shoes cause foot and back damage, if the marriage, or other, markets that match men and women compensates women fully for the utility gain to their husbands or other companions from their wearing high heels. This behavior is efficient even when it lowers the relative attractiveness of other women, including women who also wear high heels." (see Becker and Murphy, 2000, ch. 8). In fact, when women decide to wear high heels they trade-off the cost of wearing high heels for the utility gain they obtain from getting better husbands. Thus, wearing high heels can be understood as an equilibrium outcome of a game where women compete with each other for the best available partners.

Only a few contributions in the economics literature on human capital and labor markets have incorporated concerns for relative ranking. In particular, Moen (1999) studies

[4] See Postlewaite (1998).

[5] Conspicuous consumption (or "Veblen effects") exists when consumers are willing to pay a higher price for a functionally equivalent good (see Bagwell and Bernheim, 1996).

the incentives to invest in human capital in a model with labor market frictions and unemployment. In his model, an unemployed worker's chances of getting a job depends on his human capital relative to that of other unemployed workers because firms prefer to hire the most productive applicant due to rent sharing between them and the workers. Relative ranking affects the job finding rate and, as a result, there is a rat-race between unemployed individuals competing for job positions. Because wages are assumed to be determined by rent sharing between firms and workers (that is, the gains from education will not fully accrue to the workers in the form of higher wages) excessive competition might lead to inefficient overinvestment in human capital.

The most related contribution to this paper is a recent paper by Hopkins and Kornienko (2006). They study the effects of inequality in a tournament model where individuals compete for different rewards. Individuals, given their resources, make a simultaneous investment and output decision and then each individual is rewarded according to her relative position. The authors also emphasize the differential effect of inequality of resources and of inequality of rewards on individual equilibrium choices. While our main focus is on the relationship between inequalities of opportunities and wages and aggregate efficiency, theirs is on how changes in inequality of resources and rewards affect welfare for different segments of the population. In particular, they find that more inequality of resources lowers utility for agents in the middle and upper parts of the distribution, whereas an increase in the inequality of resources leads to lower utility for the relatively poor agents in society.[6]

3. A simple illustration: The 2 agents - 2 firms model.

This section presents a simple model with two firms and two agents that captures some of the main results that will be presented in the next section of the paper.

3.1. Firms

Let us assume that there are two firms, l and h, that produce a single homogeneous good, q_j, using a production function that combines technology and human capital as follows:

$$q_j = a_j * h_j, \tag{1}$$

[6] Galí and Fernandez (1999) also develop a tournament model of competition for places at college but their main interest was to compare the efficiency of two different mechanisms in allocating rewards: markets vs. tournaments.

where: $a_j > 0$ is the technology used by firm $j = \{l, h\}$. Assume, without loss of generality, that $a_h > a_l$. h_j is the human capital of the individual hired by firm j. Furthermore, we assume that each firm hires only one individual.[7]

Firms pay their workers their marginal product per unit of human capital employed in production. That is, firm l pays the worker it hires $w_l = a_l$ per unit of human capital and firm h pays the worker it hires $w_h = a_h$ per unit of human capital employed in the production process.

In this framework job positions differ in their payments because different firms operate with different technologies. The assumption that the production technology is linear in human capital greatly simplifies the analysis and also allows us to isolate the standard effect of inequality in the distribution of human capital on aggregate production efficiency that works through Jensen's inequality (see Mejía and St-Pierre, 2007).[8],[9] That is, if the amount of output produced is a concave function of human capital then a more unequal distribution of this factor of production across individuals would reduce aggregate production efficiency.

Because technologies are complementary to human capital in the production process, the firm operating with the most advanced technology would like to hire the individual with the highest human capital available in the labor market.[10] That is, we assume that firms rank individuals in the human capital dimension and that they make job offers to the individual with the highest human capital available in the job market. We will also assume that there is a perfectly assortative matching and that there are no search costs.

3.2. Individuals

There are two individuals with endowments of the complementary factors to the schooling process equal to θ_p and θ_r, respectively. θ_i can be thought of as a combination of all factors that complement individual's effort in the educational process, such as parental education, school and teacher quality, etc. (see Mejía and St-Pierre, 2007). Without loss of generality

[7] One can also think about one firm that has two available job positions, each operating with a different technology.

[8] This assumption also implies that the distribution of wages is independent of the distribution of human capital in the economy, which greatly simplifies the analysis and allows us to isolate changes in the distribution of returns to human capital accumulation from changes in the distribution of endowments.

[9] With any production function where human capital and technology are complements in production the analysis that follows would go through.

[10] Or, alternatively, in the case of one firm with two job positions that operate with different technologies, the firm would prefer to match the individual with the highest human capital to the job position with the advanced technology.

assume that $\theta_r \geq \theta_p$. That is, individual r (the rich individual) has a larger (or equal) endowment of the complementary factors than individual p (the poor individual).

Individuals accumulate human capital combining effort and the complementary factors to the schooling process according to the following human capital production function:

$$h = h(e, \theta), \tag{2}$$

where e stands for effort and θ for the endowment of the complementary factors.

Assumption $A1 : h(\cdot, \cdot)$ is differentiable, $h_e(\cdot, \cdot) > 0$, $h_\theta(\cdot, \cdot) > 0$, $h_{ee}(\cdot, \cdot) < 0$, $h_{\theta\theta}(\cdot, \cdot) < 0$, $h_{e\theta}(\cdot, \cdot) > 0$, and $h(0, \cdot) = 0$.

According to $A1$, human capital is an increasing and strictly concave function of both effort and the complementary factors, and the marginal effect of effort on the accumulation of human capital is increasing in the complementary factors. In other words, effort is complementary to the endowment of the complementary factors in the production of human capital. Also, effort is strictly necessary for the accumulation of human capital.

Each individual i maximizes a utility function that depends positively on consumption and negatively on effort. Furthermore we assume that the utility function is separable in the two arguments.[11] Each individual's problem is:

$$\underset{\{e\}}{Max} \quad U(c, e) = u(c) - v(e) \tag{3}$$

Assumption $A2 : u(c)$ and $v(\cdot)$ are differentiable, $u'(\cdot) > 0$, $u''(\cdot) \leq 0$, $v'(\cdot) > 0$, $v''(\cdot) > 0$ and $\lim_{\theta \to +\infty} v'(\theta) = +\infty$

Consumption equals income which, in turn, is equal to the expected wage per unit of human capital times the stock of human capital accumulated that the individual brings to the labor market. That is, consumption equals the expected wage times the amount of human capital that an individual brings to the market, $E(w) * h$.

Before going to the job market both individuals accumulate human capital and they know that the two firms will rank them in the human capital dimension. As a result , individual i's perceived probability of being hired by the advanced technology firm (that is, her expected wage) is a function of her human capital, h_i, and the human capital of individual j, h_j. Individual i's expected wage is given by:

[11] This is perfectly equivalent to a situation where consumption and leisure are the only arguments in the utility function and where leisure time is sacrificed when time and effort are devoted to the accumulation of human capital.

$$E(w_i) = p(h_i, h_j) * w_h + (1 - p(h_i, h_j)) * w_l, \tag{4}$$

where $p(h_i, h_j)$ is the probability, as perceived by individual i, of being hired by the firm that pays the high wage (that is, the firm that operates with the advanced technology).

Assumption $A3 : p_{h_i} > 0,\ p_{h_j} < 0,\ p_{h_i h_i} < 0$.

$A3$ says that the probability of being hired by the advanced technology firm for individual i increases as her human capital increases and decreases with the human capital of the other individual. Furthermore, this probability is strictly concave on h_i.

Assuming, again, without loss of generality, that $u(c) = c$, individual i takes individual j's effort as given and chooses her own effort to maximize her utility.[12] Individual i's problem is:

$$\max_{\{e_i\}} E(w_i) h(e_i, \theta_i) - v(e_i). \tag{5}$$

The first order condition of individual i's problem is:[13]

$$\frac{\partial p(h_i, h_j)}{\partial h_i} \frac{\partial h_i(\hat{e}_i, \theta_i)}{\partial e_i} (w_h - w_l) h(\hat{e}_i, \theta_i)) + E(w_i) h_{e_i}(\hat{e}_i, \theta_i) - v'(\hat{e}_i) = 0. \tag{6}$$

The second and third term on the left hand side of equation 6 are the standard terms in models of human capital accumulation *á-la*-Becker - Ben-Porath: the direct marginal benefit and cost from exerting one extra unit of effort in the accumulation of human capital. The first term captures how an extra unit of time and effort allocated to the accumulation of human capital affects the probability of being hired by the firm that pays the high wage (that is, the firm operating with the advanced technology). In other words, when firms pay different wages because, for instance, they operate with different technologies, individuals have an extra incentive to invest time and effort in the accumulation of human capital to increase the probability of being hired for the best available job.[14]

[12] In other words we assume that both agents make human capital investment decisions simultaneously. That is, they play a Nash-Cournot game of competition for the best available jobs.

[13] Assumptions A1 through A3 guarantee that the maximization problem in equation 5 has a unique and interior solution and that the first order condition in equation 6 is sufficient.

[14] Standard models of human capital accumulation do not incorporate this effect because they implicitly assume that all available jobs operate with the same technology. As a result, there is no incentive for competition between applicants as the wage rate per unit of human capital is the same in all available jobs.

3.3. Labor market equilibrium and comparative statics results

A Nash equilibrium of the game of competition for jobs is a pair of strategies $\{e_r, e_p\}$ that satisfy the first order conditions for both agents, r and p, in equation 6. These two first order conditions describe the reaction function (the choice of effort) of each agent to every possible choice of effort by the other agent.

Before proceeding it is worth specifying a benchmark case where individuals do not take into account the effect of effort on the probability of being hired by the firm operating with the high technology (the first term in equation 6). In the benchmark case individuals either take as given the probability of being hired by the firm operating with the advanced technology, or, alternatively, take as given the expected wage. The important point of setting up the benchmark case is that individuals are not able to affect the probability of being hired by the advanced technology firm by exerting more effort. In the benchmark case the first order condition is:

$$E(w_i)h_{e_i}(e_i^*, \theta_i) - v'(e_i^*) = 0, \tag{7}$$

where $E(w_i)$ is taken as given by individual i.

Proposition 1: Effort and hence human capital accumulation are higher when individuals compete for job positions than in the benchmark case where there is no competition.

Proof : If $p_{e_i} = \frac{\partial p(h_i,h_j)}{\partial h_i}\frac{\partial h_i(e_i,\theta_i)}{\partial e_i} > 0$, that is, if the probability of individual i being hired by the firm operating with the advanced technology increases as her human capital increases (i.e. as his effort increases), then $p_{e_i}(w_h - w_l)h(\hat{e}_i, \theta_i)) > 0$ and, using equation 6, $(p_h w_h + (1 - p_h)w_l)h_{e_i}(\hat{e}_i, \theta_i) - v'(\hat{e}_i) < 0$. However, in the benchmark case, $(p_h w_h + (1 - p_h)w_l)h_{e_i}(e_i^*, \theta_i) - v'(e_i^*) = 0$ and so it must be that $\hat{e}_i > e_i^*$ if the function $(p(h_i, h_j) * w_h + (1 - p(h_i, h_j)))h(e_i, \theta_i) - v(e_i)$ is strictly concave in e_i, as it is by assumptions $A1$ through $A3$.

Intuitively, when there is competition between individuals for the best available job positions they will exert more effort because they have an extra incentive to accumulate human capital beyond the standard marginal benefit (the second term in equation 6). This extra incentive is the marginal increase in the probability of being hired by the firm operating with the advanced technology that results form an extra unit of time and effort allocated to the accumulation of human capital.

Proposition 2: Higher inequality in the distribution of the complementary factors decreases average human capital in the economy. The decrease in average human capital as inequality increases is larger when individuals compete for jobs than in the benchmark case.

Proof : Define the average endowment of the complementary factors as $\overline{\theta}$, and let $\theta_r = \overline{\theta} + \delta$, and $\theta_p = \overline{\theta} - \delta$. The parameter δ captures inequality in distribution of the complementary factors. With this definition, the larger is δ, the larger is inequality in the distribution of the complementary factors. From $A3$ and the assumption that $h_{e\theta} > 0$ from $A1$, $\left|\frac{\partial p(h_p, h_r)}{\partial \delta}\right| > \frac{\partial p(h_r, h_p)}{\partial \delta}$, where $\frac{\partial p(h_p, h_r)}{\partial \delta} < 0$.

Because the probability of being hired by the advanced technology firm is a strictly concave function of effort and the endowment of the complementary factors and effort are complements in the accumulation of human capital, a higher degree of inequality in the distribution of endowments reduces aggregate effort invested in the accumulation of human capital. That is, when inequality increases, the probability perceived by the poor individual of being hired by the firm with the advanced technology decreases more than the same probability perceived by the relatively rich individual increases. This result follows directly from Jensen's inequality after noticing that $p_{e_i e_i} < 0$.

Proposition 3: As the difference between wages in the two available job positions $(w_h - w_l)$ increases, average human capital in the economy increases. That is, a larger difference in wages (i.e. the technologies employed by the two firms) increases the incentives to exert more effort and accumulate more human capital.

Proof : The results follows directly from the first order condition (equation 6) by noticing that the larger is $w_h - w_l$, the larger is the return from exerting effort that is associated with the increase in the probability of being hired by the advanced technology firm (first term of equation 6).

Notice that inequality of endowments and inequality of returns (wages) affect differently the incentives to compete for the best available job positions. While more inequality of endowments disincentives competition, more wage inequality does the opposite.[15]

In order to have some sense of the magnitude of the effect of inequality in the endowments of the complementary factors, and of inequality in returns, on effort and human capital accumulation, Figure 1 (a) and (b) present the results obtained from the calibration of the model presented above.[16] Effort and hence human capital accumulation are higher

[15] These results are in line with those obtained by Hopkins and Kornienko (2006).

[16] We use the following functional forms for the calibration of the model: $h(e_i, \theta_i) = Ae_i^{\alpha}\theta_i^{1-\alpha}$, with $0 < \alpha < 1$, $p(h_i, h_j) = \frac{h_i}{h_i + h_j}$, and $e_i = \frac{e_i^2}{2}$. Note that these functional forms satisfy assumptions $A1$ through $A3$. We set $A = 1$ and $\alpha = 3/4$. The results presented in Figure 1 (a) and (b) are robust to large variations of these parameters.

when individuals compete for job positions than in the benchmark case (Proposition 1).[17] This is seen in Figure 1a by comparing the two lines for any given level of endowment inequality. Note also from this figure that as inequality increases average human capital in the economy decreases, but in the case of competition for jobs it decreases faster (Proposition 2). Figure 1b shows how average human capital changes as the difference between wages in the two available job positions increases for a given level of endowment inequality. As the wage difference becomes larger, individuals have a higher incentive to compete for the high paying job position and thus exert more effort and accumulate more human capital (Proposition 3).

[INSERT FIGURES 1 (a) AND (b) HERE]

4. The General Model

4.1. Firms

Suppose that there is a continuum of firms indexed by j that produce a homogeneous good according to the following production function:

$$q_j = a_j * h_j,$$

where, as in equation 1, $a_j > 0$ is the technology used by firm j and h_j is the human capital of the individual hired by firm j. Assume that each firm hires only one individual. Furthermore, assume that $a_j \sim H(a)$.[18] There is perfect competition in the labor market so firms remunerate human capital according to its marginal product. That is, the wage rate paid by firm j is equal to a_j. Wages, therefore, are distributed according to $H(a)$.

4.2. Individuals

There is a continuum of individuals indexed by i. As in the two agents - two firms model, each individual has a given endowment of the factors that complement time and effort in the educational process, θ_i. The endowment of the complementary factors is distributed

[17] For the benchmark case we take the probability of being hired by the advanced technology firm to be the probability that would obtain if the two agents had engaged in a contest for the high paying position. Note that in this case individiduals take as given the probability that results in equilibrium but cannot affect it by exerting more effort.

[18] We assume that the CDF $H(.)$ is strictly increasing and continous.

in the population according to $G(\theta)$, with support in $[a, b]$. Human capital is accumulated (produced) using individual's effort and the complementary factors, according to $h(e, \theta)$ (same as in equation 2). The human capital production function satisfies $A1$ above.

Individuals derive utility from consumption and disutility from effort according to:

$$U(c, e) = u(c) - v(e). \tag{8}$$

The utility function in equation 8 satisfies $A2$.

4.3. Matching between firms and workers in the labor market

Following the approach of Hopkins and Kornienko (2004), if we let $F(h)$ be the distribution of human capital across individuals, individual i's ranking in the distribution of human capital will be given by:

$$\gamma F(h(e, \theta) + (1-\gamma) F^{-}(h(e, \theta)), \tag{9}$$

where $F^{-}(h) = \lim\limits_{h' \to h} F(h)$ is the mass of individuals with human capital strictly less than h,[19] and $\gamma \in [0, 1)$ is a parameter that captures the decrease in the payoff from "ties".[20]

We will assume that in hiring workers firms rank individuals and, because technologies are complementary to human capital in the production process, the firm with the most advanced technology would like to hire the individual with the highest human capital available in the market, the firm ranked second would like to hire the individual with the highest human capital available in the market (the individual who ranks second in the distribution of human capital), and so on and so forth. That is, there is a perfectly assortative matching between firms and individuals.

[19] A simpler definiton of rank would be just having $F(h)$ (as in Frank, 1985). The problem with this definition is that if all agents accumulate the same level of human capital, $\widehat{h}$, then, because $F(\widehat{h}) = 1$, all agents would have the highest ranking, and since there is a continumm of individuals, each having zero weight, an individual that increases her investment in human capital just above $\widehat{h}$ would see no increase in her ranking (see Hopkins and Kornienko, 2004).

[20] If all agents were to choose a level of human capital equal to $\widehat{h}$, then they would have ranking γ whereas if one individual chooses a level of human capital slightly greater than $\widehat{h}$ her ranking would be 1 $(> \gamma)$ (see Hopkins and Kornienko, 2004).

Recalling that $H(a)$ denotes the distribution of technologies across firms and that the wage rate is equal to the marginal product of human capital, $w_j = a_j$, then individual i's ranking in the distribution of human capital coincides exactly with her ranking in the distribution of wages in the economy. That is:

$$\gamma F(h(e,\theta) + (1-\gamma)F^{-}(h(e,\theta)) \ = H(w_i) \qquad \Rightarrow \tag{10}$$

$$R\left[\gamma F(h(e,\theta) + (1-\gamma)F^{-}(h(e,\theta))\right] = w_i, \tag{11}$$

where w_i is the wage rate per unit of human capital that individual i receives and $R = H^{-1}$ is the inverse function (the quantile function) of the CDF of a.[21]

4.4. Individuals' optimization problem

Individuals take as given other individuals' effort and choose their own effort, e, to maximize $U(c, e)$.[22] In the following, we assume that $u(c) = c$ without loss of generality[23]. As usual, the objective of an agent with endowment $\theta \in [a, b]$ is to solve the following problem:

$$\max_{e \,\in\, [e_a, +\infty]} R\left[\gamma F(h(e,\theta) + (1-\gamma)F^{-}(h(e,\theta))\right] h(e,\theta) - v(e), \tag{12}$$

where $R\left[\gamma F(h(e,\theta) + (1-\gamma)F^{-}(h(e,\theta))\right] h(e,\theta) \ \ (= w * h)$ is the level of income (and consumption) that an agent with an endowment θ will attain.

Assumption A4:

(i) $v(\cdot)$ is differentiable, $v'(\cdot) > 0$, $v''(\cdot) > 0$ and $\lim_{\theta\to+\infty} v'(\theta) = +\infty$ (from $A2$),

(ii) $h(\cdot,\cdot)$ is differentiable, $h_{e\theta}(\cdot,\cdot) > 0$, $h_e(\cdot,\cdot) > 0$,$h_\theta(\cdot,\cdot) > 0$, $h_{ee}(\cdot,\cdot) < 0$, $h_{\theta\theta}(\cdot,\cdot) < 0$ and $\lim_{e\to e_a} h_e(e,\theta) = +\infty$ for all $\theta \in (a, b]$,

(iii) $R(\cdot)$ is differentiable, $R'(\cdot) \geq 0$ and $R''(\cdot) \leq 0$.

[21] Recall that we have assumed before that the CDF $H(a)$ is strictly increasing and continous, so it has an inverse (quantile) function.

[22] That is, individuals play a simultaneous move game of competition for jobs.

[23] As long as the utility is monotonically increasing in c, the presence of (strict) concavity would only strengthen our results. DO YOU WANT TO SAY MORE???

4.5. Labor market equilibrium

A symmetric Nash equilibrium solution is a mapping $e : [a, b] \rightarrow [e_a, +\infty$ that assigns a choice of effort $e(\theta)$ for any possible endowment level θ, where $e(\theta)$ is chosen to solve the problem in equation 12. Notice that the assumption A4 is sufficient to ensure that the mapping $e(\cdot)$ is a function[24]. In the following, let $h^{eq}(\theta) \equiv h(e(\theta), \theta)$ be the equilibrium human capital mapping. The next results provide a characterization of the solution mapping $e(\cdot)$.

Proposition 4: If the solution $e(\cdot)$ exists then:
(i) $h^{eq}(\cdot)$ is strictly increasing
(ii) $e(\cdot)$ is continuous
(iii) $e(\cdot)$ is differentiable.

Proof: see the Appendix.

Proposition 5: Under $A4$, a solution function (symmetric Nash equilibrium of the game of competition for jobs) $e(\cdot)$ exists, is unique, and is characterized by the following differential equation with the initial condition $e(a) = e_a$.

$$e'(\theta) = -[R'(G(\theta))g(\theta)\frac{h(e, \theta)}{R(G(\theta))h_e(e, \theta) - v'(e)} + \frac{h_\theta(e, \theta))}{h_e(e, \theta)}] \tag{13}$$

Proof: see the Appendix.

4.6. Inequality of endowments, inequality of wages, and human capital accumulation.

As argued in the simple model presented in section 3, inequality of endowments (opportunities) and inequality of wages (returns) affect the incentives to invest time and effort in the accumulation of human capital in a different way. More inequality of opportunities disincentives competition for the best available jobs and, as a result, individuals exert less effort and accumulate less human capital in equilibrium. However, more inequality of wages, by increasing the incentives to compete for the best available job positions, induces higher competition between agents and, hence, aggregate (and average) human capital accumulation is higher.

[24] Indeed, the objective function in 12 is strictly quasi-concave by A4 and, hence, the solution to the maximization problem is always unique if it exists.